Wantirna College

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MATHEMATICAL METHODS CAS
UNIT 1
TECHNOLOGY INCLUDED EXAM
Written examination

Friday 8th June 2012

Reading time: (10 minutes)
Writing time: (75 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
Section 1 Multiple choice	*15*	*15*
Section 2 Short Answer	*10*	*37*

• Students are permitted to bring into the examination room: an approved log book, a TI n spire calculator, spare calculator batteries, a scientific calculator, pens, pencils, highlighters, eraser, sharpener and ruler.
• Students are NOT permitted to bring into the examination room: blank sheets of paper and/or whiteout liquid/tape.

Materials supplied

• Question and answer book of 8 pages.
• Additional paper is available if needed to complete an answer.

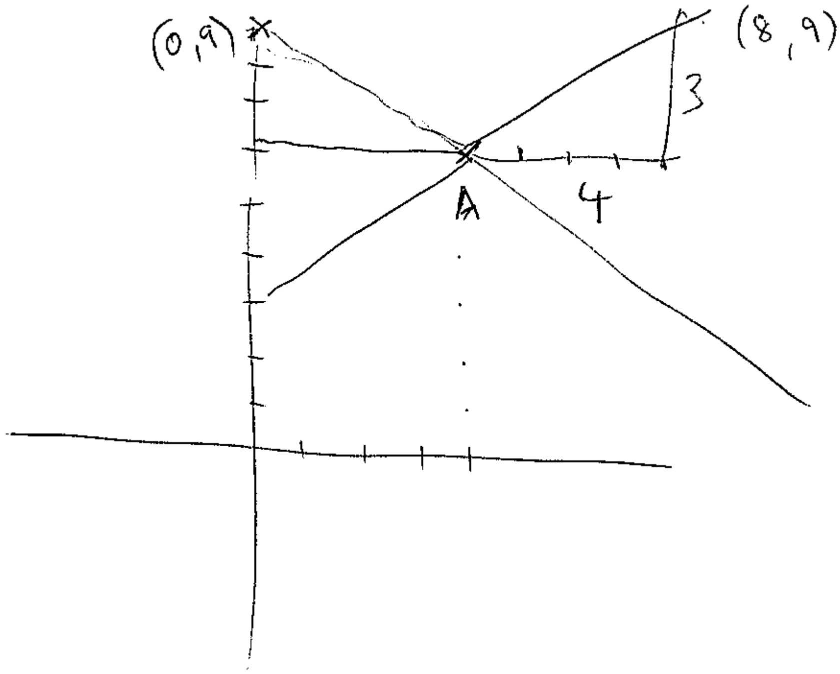
Instructions

Where a question is worth more than 1 mark, appropriate working must be shown.
Where a question is worth 2 marks, the working could be a calculator input line unless otherwise specified
Answers should be expressed in exact form unless a decimal approximation is required.

• All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic device into the examination room.

Teacher:	**AUM**	**BEA**	**TEB**	(circle your teacher's initials)



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SECTION 1 MULTIPLE CHOICE: (Answer by placing the letter corresponding to the best answer in the table at the end of multiple choice section (Q15))

1. The angle $\frac{9\pi}{4}$ is equivalent to:

A 405^0 **B** 315^0 **C** 540^0 **D** 270^0 **E** 300^0

$\sin x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

2. The solution(s) to the equation $2\sin x - \sqrt{3} = 0$ over the domain $[-\pi, \pi]$ is:

A $\frac{-\pi}{3}, \frac{\pi}{3}$ **B** $\frac{\pi}{3}$ **C** $\frac{-\pi}{6}, \frac{\pi}{6}$ **D** $\frac{-2\pi}{3}, \frac{\pi}{3}$ **E** $\frac{\pi}{3}, \frac{2\pi}{3}$

3. If $V = \frac{4}{3}\pi r^3$, then: $\sqrt[3]{\frac{3V}{4\pi}}$

A $r = \frac{V}{3} \times 4\pi^3$ **B** $r = \frac{\pi}{3}4$ **C** $r = \sqrt[3]{\frac{4\pi}{3} \times V}$ **D** $r = \sqrt[3]{\frac{3V}{4\pi}}$ **E** $r = \frac{4V\pi}{3}$

4. The expression $x^2 + 6x - 2$ factorises to:

A $(x-3+\sqrt{2})(x-3-\sqrt{2})$ **B** $(x+3+\sqrt{11})(x+3-\sqrt{11})$

C $(x+3+\sqrt{7})(x+3-\sqrt{7})$ **D** $(x-3+\sqrt{10})(x-3-\sqrt{10})$

E $(x+3+\sqrt{6})(x+3-\sqrt{6})$

$\Delta = (-3)^2 - 4 \times 2 \times -1 = 9 + 8$

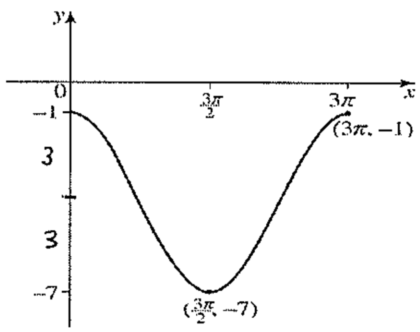
5. The discriminant of $2x^2 - 3x - 1 = 0$:

A $\sqrt{17}$ **B** 8 **C** 9 **D** -1 **E** 17

6. The effect of increasing the value k $(k > 1)$ on the graph of $y = k(x-2)^2 + 1$ is the graph is:

A Raised **B** Widened **C** Lowered **D** Shifted left **E** Narrowed

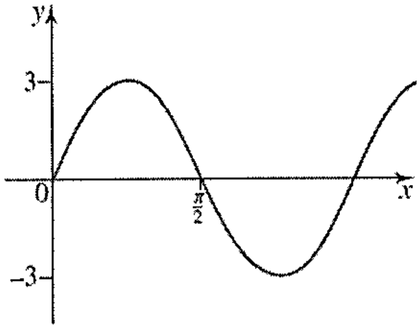
7.



The diagram above shows a graph of the form $y = a\cos nx + c$. The values of a and n respectively could be;

A. $\cancel{6}$ and $\frac{2}{3}$ **B** 3 and $\frac{2}{3}$ **C** $\cancel{-6}$ and 3π **D** $\cancel{6}$ and $\frac{3\pi}{2}$ **E** 3 and 3π

not yet covered



The equation of the function could be:

A $y = 3\sin 2x$ **B** $y = -3\sin 2x$ **C** $y = 3\sin x$ **D** $y = 3\sin\left(\frac{x}{2}\right)$ **E** $y = 3\cos 2x$

base $\theta = 30°$

S | A
T | C

9. If $\sin\theta = \frac{1}{2}$ *and* θ is in the second quadrant, then $\tan\theta$ is:

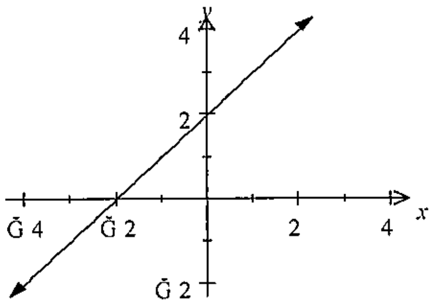
A $\frac{1}{\sqrt{3}}$ **B** $\sqrt{3}$ **C** 1 **D** $\frac{-1}{\sqrt{3}}$ **E** $-\sqrt{3}$

10. The graph with the equation $y = x^2 + 6x + 7$ has x-intercepts of:

C

A $(-3+\sqrt{7},0)$ *and* $(-3-\sqrt{7},0)$
B $(3+\sqrt{2},0)$ *and* $(3-\sqrt{2},0)$
C $(1,0)$ *and* $(-7,0)$
D $(-3+\sqrt{2},0)$ *and* $(-3-\sqrt{2},0)$
E $(-1,0)$ *and* $(-6,0)$

11. The equation of the straight line passing through the point (-4, -2) and at right angles to the one shown is:



$m = 1$ $\perp$ $m = -1$

$y - -2 = -1(x - -4)$

$y + 2 = -x - 4$

$y = -x - 6$

$x + y + 6 = 0$

? **A.** $y + x = 2$ **B** $2y + 2x - 2 = 0$ **C** $y - x + 2 = 0$ **D** $-x - y - 2 = 0$
E $-x + y - 2 = 0$

12. If the graph with the equation $y = (x+1)^2$ is shifted 2 units down and 3 units to the right, the resulting graph has the equation:

A $y = (x-2)^2 - 2$ **B** $y = (x+1)^2 - 2$ **C** $y = (x+4)^2 - 2$

D $y = (x-1)^2 + 3$ **E** $y = (x+3)^2 + 3$

$y = (x + 1 - 3)^2 - 2$

$y = (x - 2)^2 - 2$

$y = \frac{11}{-2}x + 6$

$2y = 11x + 12$

$m = \frac{6--5}{0-2} = \frac{11}{-2}$

13. The equation of the line passing through the points (-2,-5) and (0,6) is:

A $y+18-4x=0$ **B** $21+5y-2x=0$ **C** $y+9x+1=0$

D $2y-11x-12=0$ **E** $y=\frac{2x}{5}+6$

14. The function $f(x)=-2\left[\left(x+\frac{1}{2}\right)^2-3\right]$ What is range

A $(-\infty,3]$ **B** $[6,\infty)$ **C** $(-\infty,6]$ **D** $[-3,\infty)$ **E** $[-6,\infty)$

15. The graph of a parabola touches the x axis and cuts the y axis at $y=3$. A possible equation for this parabola is: $x=0$ $y=3$

A $y=3(x+1)^2$ B $y=(x+1)^2+3$ C $y=3(x-1)^2-1$

D $y=(x-1)^2+3$ E none of these

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

SHORT ANSWER QUESTIONS

1. a) Transpose the following equation to make l the subject $T=2\pi\sqrt{\frac{l}{g}}$

$g\left(\frac{T}{2\pi}\right)^2 = l$

$\div 2\pi$

sq

$\times g$

b) Solve the following equation:

$$\frac{2-x}{3}=2+\frac{2x-5}{4}$$

Solve on CAS.

$x = \frac{-1}{10}$

1+1marks

2. One day, a group of three adults and seven children went to a theme park for a total entry fee of \$184. The next day, an extra adult and two more children joined the original group at the theme park; the entry fee on the second day was \$240.

a) Define your variables and then write two equations that can be used to represent this information.

A = cost of adult

C = cost of child

$3A + 7C = 184$

$4A + 9C = 240$

b) What was the entry fee per adult and per child at the theme park?

Solve on CAS.

A = \$24

C = \$16

2+1 = 3 marks

3. **a.** Find x so that the distance between the points A (4,6) and B (x, 9) is 5 units.

$$5^2 = \sqrt{(4-x)^2+(6-9)^2}$$
$$25 = (4-x)^2 + 9$$
$$16 = (4-x)^2$$
$$\pm 4 = 4 - x$$

$$x = 4 \pm 4$$
$$= 0 \text{ or } 8$$

b. Find an equation for the line connecting A and B

If B is $(0,9)$

$$m = \frac{9-6}{0-4} = \frac{3}{-4}$$
$$y = -\frac{3}{4}x + 9$$

B $(8,9)$

$$m = \frac{9-6}{8-4} = \frac{3}{4}$$
$$y - 9 = \frac{3}{4}(x-8)$$
$$y - 9 = \frac{3}{4}x - 6 \qquad y = \frac{3}{4}x + 3$$

2+2 = 4 marks

4. A water tank is initially empty. Water then pours into the tank at a constant rate.

a) For the first ten minutes, it flows in at a rate of 20 litres per minute. The rule for the Volume (V) is $V = a\,t \quad 0 \le t \le 10$ State the value of a.

$V = 0$ at $t = 0$ at $t = 10$ $V = 20 \times 10 = 200$ L

$a = 20$

b) After this, the tank continues to fill at a rate of 15 litres per minute. How long will it take in total to fill the tank if its capacity is 1000 litres

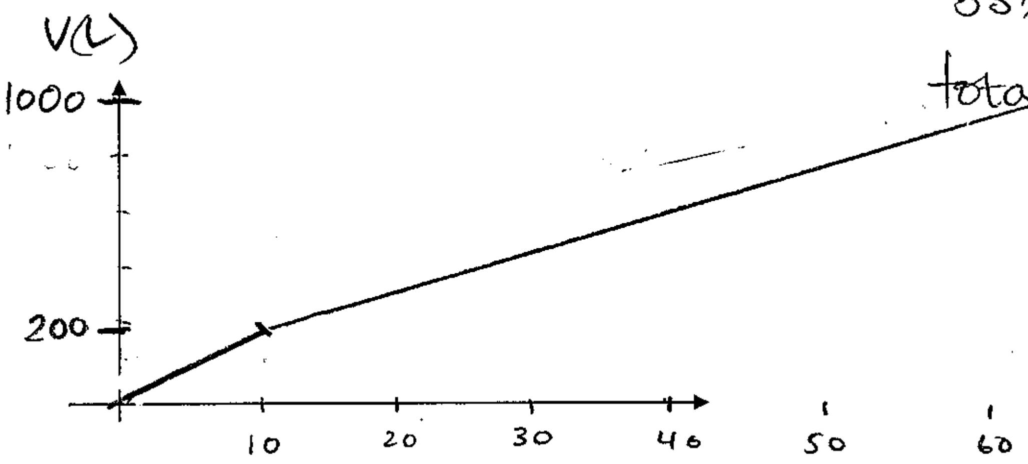
10 mins $V = 200$ L

$V = 800 = 15t \qquad t = \frac{800}{15}$

$53\frac{1}{3}$ mins

total $t = 63\frac{1}{3}$ mins

c) Sketch the graph of V against t on the axis below.



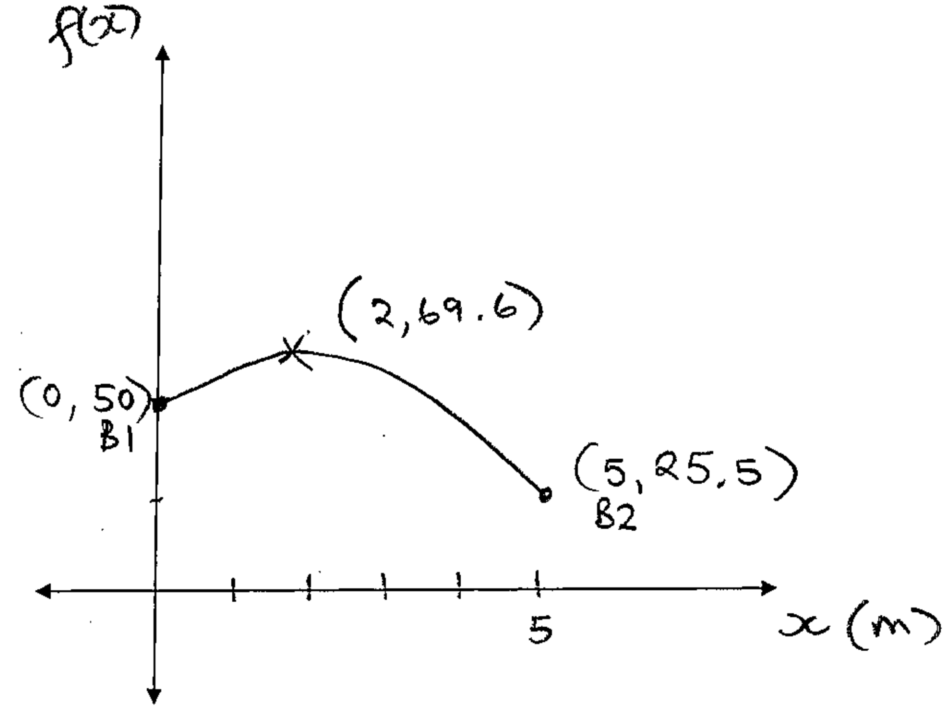
d) Write the equation to your graph as a hybrid function.

1+2+3+1 = 7 marks

$$V(t) = \begin{cases} 20t, & 0 \le t \le 10 \\ 15t, & 10 < t \le 63\frac{1}{3} \end{cases}$$

5. a) Australian FMX rider Cam Sinclair is attempting to transfer double backflip his motorcycle from building 1 to building 2 He will hit the landing ramp five metres horizontally from his take off point. The height of his trajectory can be mapped by the quadratic equation, $f(x) = -4.9x^2 + 19.6x + 50$. where x is the horizontal distance in metres.

Sketch the graph of his trajectory over the domain, indicating the heights of building 1, building 2 and the maximum height he reaches in his quest.



3 marks

6. Expand the following:

a. $(y - \frac{3}{4})^2$

$y^2 - \frac{3y}{2} + \frac{9}{16}$

b. $(2ax - 3b)(3ax + b)$

$6a^2x^2 - 7abx - 3b^2$

1+1= 2 marks

7. **a..** Factorise: $3(x-1)^2 - (x-1) - 4$

$x(3x-7)$

b. Factorise $-4x + 3x^2 - 7$=0, and then solve for x

$(x+1)(3x-7) = 0$

$x = -1 \quad x = \frac{7}{3}$

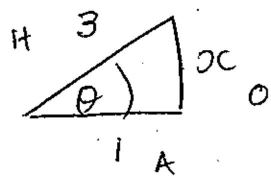
1+2 = 3 marks

8. Find all values (to 2 decimal places) of x when $\sin x = -0.65$ given $0 \le x \le 2\pi$

$$x = \sin^{-1}(-0.65)$$
$$= -0.707$$
$$= -0.71 + 2\pi \qquad \pi + 0.707$$
$$= 5.56 \quad ; \quad 3.85$$

2 marks

9. If $\cos\theta = \frac{1}{3}$ $\quad \frac{3\pi}{2} < \theta < 2\pi$ find the exact value of $\sin\theta$ and $\tan\theta$ using trigonometric identities.



$$x = \sqrt{3^2 - 1^2}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

$\sin\theta$ & $\tan\theta$ both $-$ve in 4th Q

$$\sin\theta = \frac{-2\sqrt{2}}{3}$$
$$\tan\theta = -2\sqrt{2}$$

2+2=4 marks

not yet covered

10. The number of rabbits in a national park was observed for one year.
At any time t months after observations began, the number of rabbits in thousands P can be modelled by the function $P = 2 - 0.8\sin\frac{\pi t}{6}$

a. Find the maximum number of rabbits observed in the park.

b. When would you expect this number of rabbits to be observed?

c. What is the overall variation in the numbers of rabbits in the park

d. Find the population of rabbits five months after observations began

e. For how long is the population above 2100 rabbits

1+1+1+1+3=7 marks

THE END